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OPTIMAL THERMOSTATTING OF A BUILDING

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ABSTRACT

In this paper the problem of how to design the most energy efficient thermostatting system for a building is considered. Here the given subset of rooms in a building must have given temperatures. It is proven, that if heat is supplied directly into the building then it is optimal to supply it only to the rooms with given temperatures. If air-conditioning is used for heating then it is more efficient to supply /remove heat to/from the target rooms and also to/from intermediate rooms with non-fixed temperatures.

INTRODUCTION

It is sometimes necessary to establish fixed temperatures in some rooms of a building only (we shall call them target rooms). The temperatures in other (intermediate) rooms are allowed to set freely. given temperatures of the target rooms as well as these temperatures themselves may vary, depending on the season and on the time of the day. In this paper we consider the problem of minimal energy consumption for heating/cooling of such a building. We will show that if the building is heated by direct heat supply, then for any law of heat transfer (that is, for convection heating and radiant heating) the heating shall be done by transferring heat only to thermal stated rooms. But if air-conditioning is used then the most efficient way is to supply/remove some of the energy into intermediate rooms also.

Similar problem also arises in cryogenic, where the objective is to establish a pre-set low temperature in a chamber using heat pumps. It is known, that for some laws of heat transfer, it is more efficient in this problem to use so-called active insulation. It includes an “onion ring” of chambers embedding each other, where some part of heat is removed from the central thermal stated chamber and some parts from each intermediate chamber. The temperatures in intermediate chambers are set lower than the temperature of the environment but higher than the temperature of the thermal stated chamber. The active insulation problem was first considered in [1], and then generalized in [2]. In [2] it was shown for which laws of heat transfer active insulation leads to energy savings.

In this paper we consider the problem of optimal thermal stating for a general structure building which includes a number of interconnected rooms. We consider two versions of this problem:

(A) The problem of optimal heating of this building (heat supply via electric, gas, water or air heating).

(B) The problem of optimal air-conditioning of this building (heating or cooling using the cycle of refrigerator or heat pump).

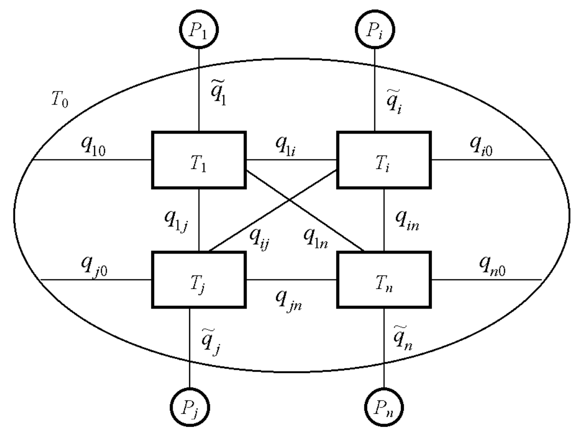

Figure 1. General structure of a building.

The structure of this building is shown in Figure 1, where the following notations are used:

T_i – is the temperature of the i-th room (i=0,1,…,n) [^0C];

$\alpha_{ij}(T_i,T_j)$ -is the heat transfer coefficient between i-th and j-th room, which can depend on the temperatures in these rooms ($\alpha_{ji}=\alpha_{ij}\geq 0$), [$Bt/^0C$];

$q_{ij}=\alpha_{ij}(T_i,T_j)(T_j-T_i)$ - is the heat flux from the i-th room to the j-th room, [Bt];

$q_{i0}=\alpha_{i0}(T_i,T_0)(T_0-T_i)$ - is the heat flux from the i-th room to the environment with the temperature T_0, [Bt];

$\tilde{q}_i$ is the heat flux, supplied (removed) to/from i-th room, [Bt]. We assume that the sign of this flux is positive if the heat is supplied to the i-th room.

Problem formulation: Assume that the temperatures of m rooms $T_1,\ldots T_m$ $(m < n)$ and the temperature of the environment T_0 are fixed. It is required to find such heat fluxes $\tilde{q}_i$ (i=1,…,n) that the total amount of heat supplied (for the problem A) or the combined power used to drive heat pumps and refrigerators (for the problem B) is minimal.

MINIMIZATION OF ENERGY CONSUMPTION FOR DIRECT HEATING

Let us write down the formally the problem of minimization of total heat supplied. This problem arises when heating system is designed for a building where the set of rooms where the temperatures are required to be fixed as well as the temperature of the environment T_0 changes during different seasons and/or during different time of the day.

The optimality criterion here is

$$I_A = \sum_{i=1}^{n} \tilde{q}_i \to \min \tag{1}$$

subject to the heat balance

$$\sum_{j=1}^{n} q_{ij}(T_i, T_j) + \tilde{q}_i = 0, \qquad i = 1,\ldots,n, \tag{2}$$

constraints on the heat fluxes

$$\tilde{q}_i \geq 0, \qquad i = 1,\ldots,n, \tag{3}$$

and constraints imposed on the temperatures of the thermal stated rooms

$$T_i = T_i^0 > T_0, \qquad i = 0,\ldots,m. \tag{4}$$

This problem can be simplified, by eliminating the condition (2) and rewriting the objective function as

$$I_A = \sum_{i=0}^{n} \sum_{j=0}^{n} q_{ij}(T_i, T_j) \to \max \tag{5}$$

subject to constraints

$$\sum_{j=0}^{n} q_{ij}(T_i, T_j) \leq 0, \qquad i = 1,\ldots,n. \tag{6}$$

The unknown variables in this problem are the temperatures of the intermediate room T_i (i=m+1,…,n).

Let us write down the Lagrange function of the problem (5), (6)

$$L = \sum_{i=0}^{n} \sum_{j=0}^{n} q_{ij}(T_i, T_j)(1 + \lambda_i) \tag{7}$$

Its optimality conditions follow from the Kuhn - Tucker theorem

$$\frac{\partial L}{\partial T_i}\delta T_i = (1 + \lambda_i)\sum_{j=0}^{k} \frac{\partial q_{ij}(T_i, T_j)}{\partial T_i}\delta T_i \leq 0, \qquad i = m+1,\ldots n, \tag{8}$$

$$\lambda_i \leq 0, \qquad \sum_{j=0}^{n} q_{ij}(T_i, T_j)\lambda_i = 0. \tag{9}$$

Here δT_i is a feasible variation of the temperature T_i.

The Slater's conditions (9) require that $\lambda_i = 0$, if $\sum_{j=0}^{n} q_{ij}(T_i, T_j) < 0$, and $\lambda_i < 0$ if $\sum_{j=0}^{n} q_{ij}(T_i, T_j) = 0$.

Since $\sum_{j=1}^{n} q_{ij}(T_i, T_j) < 0$ for the rooms to which the heat is supplied ($\tilde{q}_i \geq 0$), and $\frac{\partial q_{ij}}{\partial T_i} < 0$ for all j, from the conditions (8), (9) it follows that the heat shall be supplied only to the thermal stated rooms only (where the temperatures are given) and shall not be supplied to the intermediate rooms. The optimal values of heat fluxes $\tilde{q}_i$ ($i = 1,\ldots,m$) is uniquely determined by the heat balance equations (2) which take the following form

$$\sum_{j=0}^{n} q_{ij}(T_i, T_j) + \tilde{q}_i = 0, \qquad i = 1,\ldots,m \tag{10}$$

$$\sum_{j=0}^{n} q_{ij}(T_i, T_j) = 0, \qquad i = m+1,\ldots,n \tag{11}$$

$$T_i = T_i^0, \qquad i = 0,\ldots,m. \tag{12}$$

The conditions (10)-(12) allow us to find the fluxes $\tilde{q}_i$ ($i = 1,\ldots,m$) and (n-m) temperatures in intermediate rooms. For the Newton (linear) law of heat transfer, the heat transfer coefficients α_{ij} are constant and the problem (4)-(6) becomes the linear programming problem, and the conditions (10)-(11) turn out to be the set of (n-m) linear equations. The solution of this set of equations completely determines the optimal values of fluxes $\tilde{q}_i$. If one of the fluxes $\tilde{q}_i$ turns out to be negative then no optimal solution exists for the heating problem (A) and it is necessary to utilize air-conditioning for heating.

MINIMIZATION OF ENERGY CONSUMPTION FOR HEAT PUMP/AIR CONDITIONING BASED HEATING/COOLING

The problem of minimization of the combined energy used by air-conditioning system takes the following form

$$I_B = \sum_{i=1}^{n} P_i \to \min \tag{13}$$

subject to conditions (2), (4). We denote the efficiencies of heat pumps as $r_i = \frac{\tilde{q}_i}{P_i}$. These efficiencies depend on the design of the pump (the heat transfer coefficients in the heater and refrigerator k_0 and k_i), the form of the cycle, the temperatures on the hot and cot side of the cycle T_0 and T_i and on the power used P_i. The reversible estimate of the heat efficiency of the heat engine does not depend on P_i

$$r_i = \frac{T_i}{T_i - T_0}. \tag{14}$$

Here and later we measure temperatures in Kelvins.
The more accurate lower estimate for the efficiency of a heat pump and refrigerator cycle, which takes into account the irreversibility of heat transfer was obtained in [3], [4]. For the Newton law of heat transfer with the heat transfer coefficient k_0 for the heat removal from the environment and k_i for the heat supply into the room this estimate has the following form [3]

$$r_i(T_0,T_i,P_i) = 1+\frac{1}{2P_i}\left[\sqrt{P_i^2+\frac{k(T_i+T_0)}{2}P_i+\frac{k^2(T_i-T_0)^2}{16}}-P_i-\frac{k(T_i-T_0)}{4}\right], \tag{15}$$

here $\bar{k}_i = \frac{4k_ik_0}{(\sqrt{k_i}+\sqrt{k_0})^2}$ is the equivalent heat transfer coefficient.
Let us rewrite the condition (2) in the following form

$$\sum_{j=0}^{n} q_{ij}(T_i,T_j) + P_i r_i(T_0,T_i,P_i) = 0, \quad i = 1,\ldots,n \tag{16}$$

In the problem (13), (16), (4) the unknown variables are powers $P_i \geq 0$ ($i = 1,\ldots,n$) and the temperatures of the intermediate rooms T_i ($i = m+1,\ldots,n$).

If $\sum_{j=0}^{n} q_{ij} < 0$, then the air-conditioner for the i-th room operates as a heat pump and its r_i has the form (15). If $\sum_{j=0}^{n} q_{ij} > 0$, then it operates as a refrigerator, with $T_i < T_0$. In this case the reversible efficiency is

$$\breve{r}_i^{\,0} = \frac{T_i}{T_i - T_0} = -r_i^{\,0}.$$

For an irreversible cycle the efficiencies r_i in the conditions (16) and all equations which follow from them should be replaced with

$$\breve{r}_i = -r_i(T_i,T_0,P_i)-1). \tag{17}$$

Note that the temperatures T_0 and T_i in equation (17) changed places. The equality (17) follows from the known relation between the efficiency of refrigerating cycle and the efficiency of heat pump [5].

The Lagrange function of the problem (13), (14), (4) has the form $L = \sum_{i=1}^{1}\left\{P_i\left[1+\lambda_i r_i(T_0,T_i,P_i)\right]+\lambda_i\sum_{j=0}^{n} q_{ij}(T_i,T_j)\right\}$ which yields the following optimality conditions

$$\frac{\partial L}{\partial P_i} = 0 \rightarrow r_i(T_0,T_i,P_i) + P_i\frac{\partial r_i}{\partial P_i} = -\frac{1}{\lambda_i}, \quad i = 1,\ldots,n. \tag{18}$$

$$\frac{\partial L}{\partial T_\nu} = 0 \rightarrow P_\nu\frac{\partial r_\nu}{\partial T_\nu} + \sum_{j=0}^{n}\frac{\partial q_{\nu j}}{\partial T_\nu} + \sum_{\substack{i=1,\\ i\neq\nu}}^{n}\frac{\partial q_{i\nu}}{\partial T_\nu} = 0,$$

$$\nu = m+1,\ldots,n \tag{19}$$

These conditions jointly with the conditions (16) and expressions (15),(17) determine the unknown variables.
If a reversible efficiency estimate is used then the problem is simplified and the system (16), (18), (19) leads to the following equations

$$P_i = -(1-\frac{T_0}{T_i})\sum_{j=0}^{n} q_{ij}(T_i,T_j), \quad i = 1,\ldots,n \tag{20}$$

$$\lambda_i = -1+\frac{T_0}{T_i}, \qquad i = 1,\ldots,n \tag{21}$$

$$\lambda_\nu\sum_{\nu=0}^{n}\frac{\partial q_{\nu j}}{\partial T_\nu} + \sum_{\substack{i=1,\\ i\neq\nu}}^{n}\frac{\partial q_{i\nu}}{\partial T_\nu} - P_\nu\lambda_\nu\frac{T_0}{(T_\nu - T_0)^2} = 0,$$

$$\nu = m+1,\ldots,n \tag{22}$$

Thus the temperatures of the intermediate rooms are

$$\frac{T_\nu}{T_0 - T_\nu}\sum_{j=0}^{n}\frac{\partial q_{\nu j}}{\partial T_{\nu i}} + \sum_{\substack{i=1,\\ i\neq\nu}}^{n}\frac{T_i - T_0}{T_i}\frac{\partial q_{i\nu}}{\partial T_\nu} + \frac{T_0}{T_\nu^{\ 2}}\sum_{j=0}^{n}\frac{\partial q_{\nu i}}{\partial T_i} = 0,$$

$$\nu = m+1,\ldots,n. \tag{23}$$

This system of equations allows us to find all the temperatures, because all the temperatures for $i \leq m$ are fixed (see (12)). After finding the temperatures the powers can be found from the conditions (20) for all $i = 1,\ldots,n$.

Example

Consider the building shown in Figure 2. The temperatures are $T_0 = 253K$ and $T_1 = 293K$ and the heat transfer coefficients are $K_0 = K_1 = K_2 = 3000\frac{Bt}{K}$ and $\alpha_{10} = \alpha_{20} = 94.08\frac{Bt}{K}$ and $\alpha_{12} = \alpha_{21} = 180\frac{Bt}{K}$. It is required to find the temperature T_2 in the second room and the powers of heat engines.

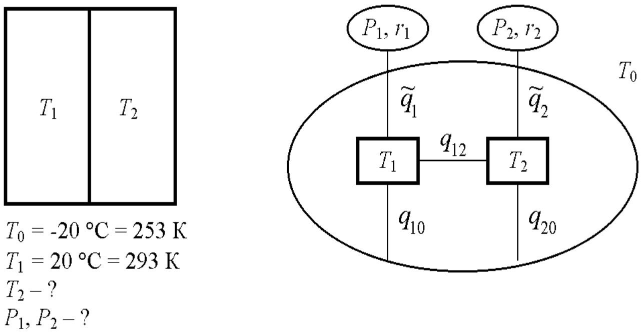

Figure 2. The plan and the computational structure of the building used in Example.

The problem of minimal energy used to drive heat pumps has the following form here

$$I = P_1 + P_2 \to \min$$

subject to hat balance

$$q_{10}(T_1, T_0) + q_{12}(T_1, T_2) + P_1 r_1(T_0, T_1, P_1) = 0,$$

$$q_{20}(T_2, T_0) + q_{21}(T_2, T_1) + P_2 r_2(T_0, T_2, P_2) = 0,$$

Now power can be expressed in term of T_2 as

$$P_1(T_2) = 1.6\frac{848560349 - 4468950T_2 + 5625T_2^{\,2}}{76737 - 50T_2}$$

$$P_2(T_2) = 0.48\frac{1769323311113 - 1313982242T_2 + 2436457T_2^{\,2}}{4267T_2 - 318926}$$

Thus, the optimality criterion I depends only on T_2 only and attends its minimum at $T_2 = 282K$.

Substitution of the obtained temperature T_2 into the expressions for the powers yields $P_1 = 910.36Wt$ and $P_2 = 79.32Wt$ ..

CONCLUSION

In this paper we demonstrated that for any law of heat transfer if the building is heated by direct supply of heat (electrical heating, heating using hot water/air, natural gas heating) then it is most energy efficient to supply heat only into the set of rooms where the temperatures are fixed. The temperatures in the intermediate rooms are allowed to set up freely and are determined by the conditions of heat transfer.

If air-conditions are used for heating/cooling then it is most efficient to utilize some power to establish some optimal temperatures in the intermediate non-target set of rooms.

The obtained formulas allow us to estimate the lower bound on the total energy consumption for thermal stating of the building.

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